DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

STUDY GUIDE

Program of Postgraduate Studies

Mathematics and its Applications

Contents

STUDY GUIDE

1. The Department of Mathematics and Applied Mathematics

The Department of Mathematics and Applied Mathematics was created in June 2013 by the merge of the Department of Mathematics (founded in 1977) and the Department of Applied Mathematics (founded in 1999). It is both the oldest and the youngest department of the University of Crete and the largest of the Faculty of Science and Technology.

The **Department of Mathematics** admitted students for the first time in the academic year 1977-1978 and together with the Department of Philology were the first departments to operate at the University of Crete. The Department created a long tradition of excellent university teaching accompanied by significant research results in the mathematical science. From its early years of operation, it made its mark on the map of higher education. It established a flexible program of undergraduate studies and was the first of the Departments of Mathematics in Greece to operate, as early as in 1984, an organized Postgraduate Studies Program leading to the acquisition of an M.Sc. degree or even a PhD dissertation. The Department has pioneered the implementation of international practices, such as the organization, in the year 2000, of its external evaluation.

The **Department of Applied Mathematics** was founded by members of the Department of Mathematics, in 1999, with the aim of developing the applications of Mathematics in Greece. It attracted accomplished and dynamic young researchers from Europe and America and soon developed excellent education and research in applied Mathematics. It secured important competitive research projects and activities and introduced innovative ways of teaching and innovative courses in the program of studies.

The unified Department continues the excellent academic tradition of the two Departments, as demonstrated in their external evaluations by H.A.H.E. (Hellenic Authority of Higher Education). The quality of teaching is comparable to that of many world-leading Universities, where almost all staff have studied and worked. Faculty members are active in research, have developed international collaborations and publish their research in high-impact journals.

2. Postgraduate Program “Mathematics and its Applications”

The Department of Mathematics and Applied Mathematics at the University of Crete operates a MSc Program titled "Mathematics and its Applications." The MSc focuses on Mathematics and their applications. Its purpose is to promote knowledge and develop research in areas of Mathematics and its applications, as specified by the three specializations of the program.

The program will be staffed primarily by the faculty members of the Department of Mathematics and Applied Mathematics of the University of Crete. Graduates of the awarded MSc Degree are expected to have a high-level expertise in Mathematics and its applications, with the potential to continue their studies towards obtaining a Ph.D. or to be employed in Universities and Research Centers as specialized research and technical personnel. They will also be qualified for employment in education, public services, private businesses, and organizations.

The Department of Mathematics and Applied Mathematics collaborates closely with the Foundation for Research and Technology (F.O.R.T.H.) and other recognized Research Centers. Graduate students of the MSc in Mathematics and its Applications can participate in research programs at F.O.R.T.H. and other recognized Research Centers, aiming to gain training in specialized topics and conduct research work related to their MSc thesis.

3. Admission of students to the program

A. The MSc accepts, graduates from Departments of Schools of Sciences, Engineering Schools and Schools of Economics from Greek universities and equivalent recognized institutions abroad. The number of students admitted to the program each year is set at a maximum of twenty (20) and a minimum of five (5).

The call for applications is posted on the websites of the program and the University and is announced through all appropriate means to other higher education institutions in the country. The call includes the criteria and the procedure for the selection of candidates, the deadline for submitting applications and the method of submission, as well as the contact information of the administrative support service of the Program.

B. Submission of Applications
The documents that must be submitted to the Postgraduate Studies Secretariat of the Department of Mathematics and Applied Mathematics are:

- Application for admission to the Postgraduate Program.
- Curriculum Vitae.
- Transcript of records.
- Copies of English language certificates (level B2 or higher)
- Two (2) to three (3) recommendation letters from professors who are personally familiar with the candidate's academic progress in undergraduate studies
- Any other supplementary qualifications that may be considered necessary by the candidates (e.g., additional academic degrees, scholarships (IKY), performance in exams (e.g., GRE, TOEFL), or academic competitions (Mathematical Society Competition, etc.).
- Brief description of the candidate's scientific interests.
- Copy of diploma or official certificate of completion of studies. If the application is accepted, a prerequisite for enrollment in the M.Sc. program is the submission of a copy of the undergraduate degree certificate. If the undergraduate degree comes from an institution abroad, it should be assessed and recognized by the Interdisciplinary Organization for Recognition of Academic Titles & Information (D.O.A.T.A.P.), and this procedure is conducted by the M.Sc. Secretariat in accordance with the provisions of law 4957/2022.
- For enrollment in the M.Sc. program, it is necessary to submit, if required by law, certified copies of the above-mentioned documents.

C Submission and Processing of Personal Data
Applications, recommendation letters, and relevant documents are submitted electronically through the specified University's website. Personal data is collected based on the General Data Protection Regulation 2016/679. These data remain accessible to the University of Crete for the duration of studies and even after graduation for future reference. During this period, in which personal data remains available to the University, individuals have the right to access, correct, update, restrict or object processing, and request data portability under the terms of the General Data Protection Regulation 2016/679 (E.U.).

D. Process and Selection Criteria
The evaluation of applications is carried out by a committee appointed annually by the Department Assembly and consists of faculty members of the Department. The selection criteria considered are:

1. Overall performance of the candidate in courses,
2. Grades in courses relevant to the subject matter of the M.Sc. program,

3. Interview and/or written examinations,
4. Any significant achievements during undergraduate studies,
5. Performance in any thesis or dissertation,
6. Recommendation letters,
7. Basic knowledge of the English language (B2 level – First Certificate of English),
8. Any other qualifications presented by the candidate in their application.

Further details on quantifying the above criteria will be provided in the call for applications.

E. Providing Equal Opportunities

The Department aims to provide equal opportunities in education and training. It ensures equal opportunities in the admission and completion of postgraduate studies without discrimination based on gender, colour, nationality, religion, or personal status, in accordance with Greek legislation.

4. Duration of Studies, Extensions and Interruptions of Studies

The duration of studies for the completion of the MSc in Mathematics and its Applications is set at four (4) academic terms.
Part-time study is available for working students or, exceptionally, for non-working students, with a maximum duration of eight (8) academic semesters.
An extension and/or interruption of studies (up to one year each) may be granted for serious personal reasons, upon request until the end of the second week of classes of each term, according to current legislation.

5. Recognition of Courses

Upon request of the interested student and recommendation by the Coordinating Committee, the Department may recognize certain postgraduate courses, which the student attended at a University during their undergraduate or postgraduate studies, for fulfilling the requirements of the postgraduate program of studies. This is done by determining the equivalent ECTS. If the course is part of the Department's MSc program, it will be credited with 10 ECTS, provided that these ECTS have not already been used for obtaining another undergraduate or postgraduate degree. The total number of ECTS that can be recognized cannot exceed 30 ECTS.

6. Master’s Thesis

The preparation of a master's thesis is carried out in the area of specialization. The master's thesis is supervised and guided by the Supervisor and is examined by the Examination Committee. The language of preparation and writing of the master's thesis can be either Greek or English. The master's thesis must be accompanied by a summary in the language of writing and an extensive summary in the other language.

The Coordinating Committee appoints a supervisor, after receiving an application from the candidate, which includes the proposed title of the thesis, the proposed supervisor, and a summary of the proposed thesis. For the evaluation and approval of the thesis, the Coordinating Committee forms a three-member examination committee. The supervisor is one of the three members of the committee, and all members must have the legal qualifications according to current regulations. Additionally, at least one of the members of the three-member committee must be a faculty member of the Department in a related field.

The candidate submits the thesis text to the members of the Examination Committee at least 15 days before the examination date. To be approved, the student must defend the thesis before the examination committee. The examination of the master's thesis is oral and public and follows this procedure: The candidate presents the thesis. This is followed by questions from the members of the Examination Committee and then from the audience.

The members of the Examination Committee provide any comments on the content of the thesis to the candidate. The candidate must consider the Committee's comments in the final text. Depending on the nature and extent of the modifications/improvements, the members of the committee may request to see the thesis text again or authorize the Supervisor to give final approval.

The members of the Examination Committee approve the final text and sign the evaluation report of the thesis. Finally, the candidate submits two signed copies of the thesis, one to the Graduate Studies Office of the Department and one to the University Library. If the thesis has been financially supported by a scholarship, a third copy must be submitted to the Graduate Studies Office of the Department for delivery to the sponsoring organization. Additionally, the candidate submits the final text of the thesis in electronic form to the Graduate Studies Office of the Department. Master's theses are recorded in the Digital Library of the University of Crete and are posted on the Department's website.

7. Auxiliary teaching duties

Auxiliary teaching duties include all tasks (usually supervising laboratories, tutorials, etc.) assigned by the Department within the framework of conducting courses, examinations, and all educational processes. Auxiliary teaching duties are mandatory for two semesters of the postgraduate program.

8. Evaluation of Students

Each year, the performance of students is evaluated by the Program's Coordinating Committee (C.C.). If, at the end of two semesters, a student has not successfully passed at least three (3) courses, with a grade of B- or better in at least two (2) of them, their performance is deemed unsatisfactory. In the case of unsatisfactory performance, the C.C. may recommend to the Steering Committee the student's dismissal.
Exams are conducted under the responsibility of the instructors of each course, in accordance with current regulations. The grading scale for successful performance, in descending order, is: A+, A, A-, B+, B, B-, C+, C, C-. For unsatisfactory performance: D. Alternative methods will be applied for the evaluation of students with disabilities and special educational needs.

9. Evaluation of Teaching

The evaluation of postgraduate courses and instructors by postgraduate students is conducted through the completion of questionnaires in electronic form.

10. Language of Instruction

The language of instruction for courses is Greek, and it may be English in cases where the course is attended by postgraduate students whose native language is not Greek. The language for writing the postgraduate thesis can be either Greek or English.

11. Academic Advisor

For each postgraduate student admitted to the M.Sc., the Department appoints an academic advisor, who is a faculty member of the Department. The role of the academic advisor is to assist the candidate in adapting to the postgraduate program, selecting courses, and addressing any academic issues that may arise during their postgraduate studies. The student must choose the professor/researcher, after consultation with them, with whom they intend to undertake their thesis by the beginning of the third (3rd) semester's registration and inform the Coordinating Committee (C.C.) in writing. At that point, the role of the academic advisor is transferred to the above-mentioned individual. After the appointment of the thesis supervisor by the C.C., this person also assumes the role of the academic advisor from then on.

12. Completion of Part of Studies through the Erasmus Program

Students of the postgraduate programs of the Department of Mathematics and Applied Mathematics have the opportunity to complete part of their studies abroad to attend courses through the ERASMUS Program. Attendance of courses can take place at a selected higher education institution with which the University has established a bilateral agreement, and which the interested student can find on the Department's website. Each mobility occurs in one of the 30 European countries for a period of 3-6 months with full recognition of the mobility period. In the case of mobility for studies, students must enroll in courses that correspond to the postgraduate program's curriculum, thus allowing them to earn up to 30 ECTS per semester. On the Department's website, interested students can find a description of the relevant procedure and the selection and ranking criteria.

13. Student Welfare

The Student Welfare Office supports and promotes initiatives that assist students, foster their intellectual growth, and support their personal development. It also helps them explore and experience various aspects of student life.
Students at the University of Crete have the possibility to apply for free meals at the student restaurants. The selection of students is made on the recommendation of the relevant Committee, according to the current legislation, the financial resources of the University of Crete, and the students' ranking.
Students who are not eligible for free meals can dine at the student restaurants by paying approximately €3 per meal or using 7-day and 15-day meal cards priced at €14.25 and €30.50, respectively, which include breakfast, lunch and dinner, and which are provided by the restaurants.
For more information, please visit the website: https://www.merimna.uoc.gr/index.php/el/

14. University Gym

Since 2007, the University of Crete campus in Voutes, located in the city of Heraklion, has been home to a comprehensive sports center. This facility includes an indoor gymnasium with a seating capacity of 1,080 and a 25-meter indoor swimming pool with five lanes. Additionally, the University of Crete owns a full-size football field with artificial turf in the area of Agios Ioannis Knossos.
For more information, please visit the website: https://unisport.uoc.gr/

15. Transportation from the City of Heraklion to the Voutes University Campus

By Bus: There are regular bus services operated by the Heraklion Urban KTEL connecting various neighborhoods of the city to the Voutes University Campus.
By Car: The Voutes University Campus is located 8.5 kilometers southwest of the city center. To get there, head towards the Heraklion-Moires National Road and before completing the first kilometer, turn right at the traffic light after Estavromenos. There are signs directing you to the university from there.
By Taxi: For taxi information, you can visit the following websites:
https://candiataxi.gr/
https://www.cretataxi.com/
For more information, you can visit the university's websites:
https://www.uoc.gr/university/traf/traf.html
https://visit.uoc.gr/xartes/

16. Accessibility

Externally, there are special access routes for individuals with disabilities (PWD) to all buildings on the Voutes University Campus of the University of Crete. Internally, aside from vertical transportation via common elevators, the Mathematics building features external access to lecture amphitheaters and dedicated spaces for PWD within the amphitheaters. Additionally, computer facilities have reserved desks for PWD, available on the upper level of the teaching amphitheaters. There is direct accessibility to all classrooms of the Department. Automatic door opening is also available.

17. Academic Staff

A complete list of the Academic Staff can be found here.

18. Program Requirements

A typical postgraduate course has 13 weeks of instruction. Classes are rescheduled under the responsibility of the instructors and with a relevant announcement from the administration office. For courses worth 10 ECTS, the total hours of educational activity are estimated at two hundred and fifty (250) hours per course. This includes four (4) teaching hours per week, twelve (12) hours of study and assignments per week, as well as forty-two (42) hours for exams (final and midterms), including preparation time.
Ένα τυπικό μεταπτυχιακό μάθημα έχει 13 εβδομάδες διδασκαλίας. Τα μαθήματα αναπληρώνονται με ευθύνη των διδασκόντων και με σχετική ανακοίνωση της γραμματείας. Για τα μαθήματα των 10 ECTS οι συνολικές ώρες εκπαιδευτικής δραστηριότητας εκτιμώνται σε διακόσιες πενήντα (250) ανά μάθημα. Περιλαμβάνουν τέσσερις (4) διδακτικές ώρες ανά εβδομάδα, δώδεκα (12) ώρες μελέτης και εργασιών ανά εβδομάδα, καθώς και σαράντα δύο (42) ώρες εξετάσεων (τελικές και πρόοδοι), συμπεριλαμβανομένων και των ωρών προετοιμασίας.

19. Requirements for Obtaining the M.Sc. in Mathematics and its applications

The total number of ECTS required for the award of the M.Sc. is 120. To obtain the M.Sc., the following is required:

- Attendance and successful examination in at least six (6) courses, which collectively account for 60 ECTS.
- Writing a master's thesis in the area of specialization, which is credited with 40 ECTS. Alternatively, in Specializations I and II, instead of writing a master's thesis, attendance and successful examination in additional courses amounting to 40 ECTS in total is required. In Specialization III, the master's thesis is mandatory.
- The undertaking of auxiliary teaching work (teaching assistant) for two academic semesters, which is credited with 2 ECTS per semester (4 ECTS in total).
- To complete the required ECTS credits, students must successfully attend additional courses beyond the above or participate in Thematic Activities, fulfilling the respective obligations.

Course Requirements per Specialization

In each specialization, students are required to successfully attend at least four Core Specialization Courses (CSC). Specifically:

For Specialization I:

- Four Core Specialization Courses must be chosen from the core courses of Specialization I, from at least three of the following groups of courses: [A1], [B], [C], [D1 or E].

For Specialization II:

- Four Core Specialization Courses must be chosen as follows: [A11], [A10 or A14], [A21 or A22 or A23], [A31 or A32 or A39].

For Specialization III:

- For students choosing Thematic Area IIIa "Differential Equations," the four Core Specialization Courses must be chosen as follows: [D10], [H10], [B0], [B1 or C0 or C1 or D11 or E10].
- For students choosing Thematic Area IIIb "Numerical Methods," the four Core Specialization Courses must be chosen as follows: [H10], [B0 or B1], [D10 or D11], [H11 or H12 or H13]

20. Detailed Program of Studies

(a) The curriculum consists of Core Specialization Courses (CS) as well as Elective Courses (EC). In each specialization, students are required to successfully complete at least four Core Specialization Courses. In addition to the graduate courses, students in the Postgraduate Program may participate in the following indicative thematic activities (TA) as part of their studies, fulfilling the corresponding obligations:

List of Thematic Activities			
	Name	**Participation**	**ECTS**
801.1	Seminar Courses	Optional	2
801.2	Seminar Courses	Optional	4
801.3	Seminar Courses	Optional	6
802.1	Teaching Assistance (per semester)	Optional	2
802.2	Teaching Assistance (per semester)	Optional	2
803	Supervised Study Seminar	Optional	6
804	Technical Writing in English	Optional	4

Indicatively, the program is structured as follows:

1st SEMESTER		**2nd SEMESTER**	
COURSES	**ECTS**	**COURSES**	**ECTS**
Core Specialization Course	10	Core Specialization Course	10
Core Specialization Course	10	Core Specialization Course	10
Elective Course	10	Elective Course	10
TOTAL	**30**	**TOTAL**	**30**

3rd SEMESTER		**4th SEMESTER**	
COURSES	**ECTS**	**COURSES**	**ECTS**
Elective Course	10	Thematic Activities	4
Thematic Activities	6		
Thesis	14	Thesis	26
TOTAL	**30**	**TOTAL**	**30**

(b) The table below presents an indicative list of Core Specialization Courses and Elective Courses for the three different specializations. This list, as well as the above list of Thematic Activities, may be modified by decision of the Department Assembly. It is adjusted and specified for each Academic Year by decision of the Department Assembly.

	Course Groups	BE	EC	ECTS
	Group A1			
A10	Algebra I	I, II	III	10
A11	Algebra II	I, II	III	10
A12	Algebraic Number Theory		I, II, III	10
A13	Group Representations		I, II, III	10
A14	Algebraic Geometry	II	I, III	10
A19	Topics in Algebra and Number Theory		I, II, III	10
	Group A2			
A20	Set Theory		I, II, III	10
A21	Logic	II	I, III	10
A22	Computability	II	I, III	10
A23	Algorithms and Complexity	II	I, III	10
A29	Topics in the Foundations of Mathematics		I, II, III	10
	Group A3			
A30	Combinatorics		I, II, III	10
A31	Cryptography	II	I, III	10
A32	Coding Theory	II	I, III	10
A38	Topics in Symbolic – Algebraic – Combinatorial Computations		I, II, III	10
A39	Topics in Discrete Mathematics	II	I, III	
	Group B			
B0	Real Analysis	I,III	II	10
B1	Functional Analysis	I, III	I, II	10
B2	Complex Analysis	I	II, III	10

B3	Harmonic Analysis		I, II, III	10
B4	Ergodic Theory		I, II, III	10
B9	Topics in Analysis		I, II, III	10
	Group C			
Γ0	Riemannian Geometry	I, IIIα	II, IIIβ	10
Γ1	Differentiable Manifolds	I, IIIα	II, IIIβ	10
Γ2	Algebraic Topology – Homotopy	I	II, III	10
Γ3	Algebraic Topology – Homology		I, II, III	10
Γ4	Geometry of Dynamical Systems		I, II, III	10
Γ9	Topics in Geometry – Topology		I, II, III	10
	Group D			
Δ10	Partial Differential Equations	I,III	II	10
Δ11	Partial Differential Equations – Theory of Weak Solutions	III	I, II	10
Δ12	Ordinary Differential Equations and Dynamical Systems		I, II, III	10
Δ14	Calculus of Variations		I, II, III	10
Δ15	Mathematical Theory of Fluids		I, II, III	10
Δ19	Topics in Differential Equations		I, II, III	10
	Group E			
Θ10	Numerical Analysis	III	I, II	10
Θ11	Numerical Solution of Differential Equations	IIIβ	I, II, IIIα	10
Θ12	Finite Element Methods	IIIβ	IIIα	10
Θ13	Numerical Linear Algebra and Optimization	IIIβ	I, II, IIIα	10
Θ20	Topics in Numerical Analysis		I, II, III	10
Θ30	Topics in Numerical Methods		III	
	Group F			
E10	Probability Theory	I, IIIα	II, IIIβ	10
E11	Stochastic Analysis		I, II, III	10

E18	Topics in Stochastic Analysis		I, II, III	10
E19	Topics in Probability Theory		I, II, III	10

(c) By decision of the Department Assembly, the following can be added to the list of Elective Courses: (i) graduate courses from another Graduate Program of the University of Crete, provided that they have related or complementary content to the respective courses of the present Graduate Program, (ii) graduate courses offered by a university within the country or abroad, where the graduate student is transferred as part of an educational exchange program (e.g., Erasmus), (iii) graduate-level courses which the graduate student can attend as part of a school or a series of lectures, for which ECTS credits are awarded. Additionally, exceptionally according to Article 67 of Law 4957/2022, by decision of the Department Assembly, students of the Graduate Program are allowed to attend, remotely, with synchronous teaching, graduate courses taught by faculty members of departments of the University of Crete based in Rethymno or other domestic higher education institutions. The above courses are matched with courses of the Graduate Program with similar content or can be added to the list of courses of the Graduate Program. The total number of credits that can be recognized in this way cannot exceed 20 ECTS credits.
Below are the graduate courses offered in the Graduate Program.

Course Description

(detailed outlines here)

A10: ALGEBRA I

- I. Groups: Group actions on sets. Sylow theorems. Properties of p-groups. Nilpotent groups. Solvable groups. Jordan-Holder theorem.
- II. Rings: Rings, subrings, ideals, prime and maximal ideals. Associative, prime, and irreducible elements of a ring. Principal ideal domains (PID), unique factorization domains (UFD), Euclidean domains and their relationships. Noetherian and Artinian rings. Polynomial rings. Hilbert's basis theorem and Nullstellensatz, Gauss' theorem. Primary ideals and Lasker-Noether theorem. Valuation rings and Dedekind rings.
- III. Modules: Modules, homomorphisms, and exact sequences. Free modules, basic properties. Projective, injective, and flat modules. Tensor products. Modules over a PID and the fundamental structure theorem. Applications to matrices (rational canonical form, Jordan canonical form) and abelian groups (the classification theorem).

A11: ALGEBRA II

- I. Field Extensions: Definitions. Degree of extension. Algebraic and transcendental extensions.
- II. Algebraic extensions: Finite extensions. Minimal polynomial of an element. Simple extensions. Finite generated extensions. Transitivity of algebraic extensions. Field of roots of a polynomial. Algebraic closure. Embedding extensions. Normal extensions. Separable extensions. Perfect fields. Primitive element theorem. Extensions with finite fields. Separable degree of extension. Fully inseparable extensions. Inseparable closure. Normal closure.
- III. Galois Extensions: Galois group of an extension. Polynomials and Galois groups. Fixed fields with respect to a group of automorphisms of the extension. Galois extensions. Fundamental theorem of Galois theory. Extensions with radicals. Solvability of polynomial equations with radicals. Constructible numbers. Finite fields and Galois extensions. Cyclotomic extensions. Norm and trace of elements. Discriminant. Kummer extensions. Hilbert's 90th problem.
- IV. Transcendental extensions: Transcendental elements. Transcendental bases. Noether normalization theorem. Luroth's theorem.

A12: ALGEBRAIC NUMBER THEORY

- I. Quadratic number fields.
- II. Integral dependence and Dedekind rings: Noetherian and Dedekind rings. Integral dependence. Ideal arithmetic and the final theorem.
- III. III. Norm, trace, basis, and discriminant: Norm and trace. Discriminant of an n-tuple. Free abelian groups with finite rank. Discriminant of a field and basis of its integrality.
- IV. Norm of ideals and the finite number of classes: Norm of ideals of an algebraic number field. Finite number of classes.
- V. Law of analysis and law of inversion: Application of the law of inversion to quadratic and cyclotomic fields. Hilbert's ramification theory. Laws of inversion. Discriminant theorem.
- VI. Dirichlet's unit theorem: Discrete subgroups of R^n. Normal embedding of an algebraic number field. Applications to discriminant.

A13: GROUP REPRESENTATIONS

Representations of groups. Basic definitions and examples. Equivalent representations, irreducible representations. Modules and representations, Indecomposable modules, Schur's lemma, Wedderburn's theorem, Maschke's theorem. The regular representation and its decomposition. Character theory, basic definitions and examples. Orthogonality relations, number of characters. Character values, algebraic integers and real characters. Brauer's theorem. Character table and the information it provides about the group. Applications: Abelian groups, groups of order pq and Burnside's theorem. Induced characters, Frobenius reciprocity. Normal subgroups and induced characters, Clifford's theorems. Character extensions, Gallagher's theorem.

A14: ALGEBRAIC GEOMETRY

Affine varieties. Hilbert's basis theorem and the Nullstellensatz. Polynomial functions, rational functions, and coordinate rings. Zariski topology. Sheaf of regular functions on an affine variety. Functions and morphisms. Algebraic varieties. Projective space and projective varieties. Dimension. Rational and irrational mappings. Blow up. Smooth points and singularities of a variety. Smooth varieties. Hilbert's polynomial of a projective variety. Degree of a projective variety. Intersection theory, Bezout's theorem and applications. Advanced topics including Schemes, Sheaf cohomology. Algebraic curves.

A20: SET THEORY

Intuitive notion of "set," Zermelo-Fraenkel axioms, power sets, construction of natural numbers, ordinal numbers and their arithmetic, transfinite induction, cardinal numbers, axiom of choice, axiom of continuum, "Large Cardinals" and applications, Descriptive Set Theory elements, special topics.

A21: LOGIC

Propositional calculus, truth tables, logical consequence, tautologies, formal proofs, Completeness theorem and Compactness theorem for propositional calculus, predicates, Predicate calculus, first-order languages, interpretations (models), interpretation of types and propositions, formal proofs, satisfiability of type sets, Completeness and Compactness theorems in Predicate calculus, recursive functions, Non-Completeness theorem of Arithmetic, interpretations of Set Theory and Peano Arithmetic, properties expressible in first-order languages, Many-valued Logic, elements of λ-calculus, special topics.

A22: COMPUTABILITY

Finite Automata (Finite State Machines), computability and non-computability with Finite Automata, Turing Machines, Post systems, Recursive functions, equivalence of different models of computation, Halting Problem and undecidability, algorithmic complexity and complexity measures, examples of complexity, computability in Number Theory, Algebra, and Geometry, special topics.

A23: ALGORITHMS AND COMPLEXITY

The concept of 'Problem' in Algorithm Theory, time and space complexity, databases, techniques for finding "good" algorithms, balancing, dynamic programming, sorting algorithms, complexity lower bounds, average complexity, algorithms on graphs, algorithms in Algebra and Geometry, Nondeterministically Polynomial (NP) problems, complete problems in the class NP, SAT problem, Hamiltonian problem, "clique" problem, solving systems of linear equations over integers, special topics.

A30: COMBINATORICS

Combinatorial principles using basic arithmetic operations, Generating Functions, "Pigeonhole Principle", Combinatorial principles in Set Theory, "Polya counting" using Group Theory, elements of Analytic Combinatorics, Permutation Patterns, elements of Ramsey Theory, Graph Theory, special topics.

A31: CRYPTOGRAPHY

I. Historical ciphers: Caesar cipher, monoalphabetic substitution, polyalphabetic substitution, one-time-pad.
II. Block ciphers: Feistel structure, DES, MACs.
III. Basic algebraic/number-theoretic algorithms: Euclidean algorithm, Chinese Remainder Theorem, computations in the group of integers modulo n.
IV. Public key systems: Diffie-Hellman protocol (Diffie-Hellman protocol, Diffie-Hellman computational and decision problems, discrete logarithm problem). Encryption systems (ElGamal system, RSA system, integer factorization problem, attacks and security definitions). Digital signature systems (ElGamal system, RSA system, DSA signatures, Schnorr signatures, attacks and security definitions). Additional applications (e.g., commitment systems).
V. Special topics: Time permitting (e.g., elliptic curve cryptography, knapsack cryptosystems, lattices and LLL reduction algorithm, code-based cryptosystems).

A32: CODING THEORY

I. Basic concepts in finite fields: Basic concepts and properties of finite fields.
II. Linear codes: Basic definitions, linear algebra over finite fields, encoding and decoding with syndrome.
III. Bounds: Singleton bound, definition and properties of MDS codes, Hamming bound, definition of perfect codes and Hamming codes, Gilbert-Varshamov bound.
IV. Constructions: Basic methods for constructing new codes from old ones, Reed-Muller codes.
V. Cyclotomic codes: General construction, BCH codes, classic Reed-Solomon codes.
VI. Algebraic-Geometric codes: Generalized Reed-Solomon codes, Goppa codes.

B0: REAL ANALYSIS

Measures and outer measures, Lebesgue measure and more general Borel measures, measurable functions, Lusin's theorem, integral, Lebesgue integral and Lebesgue-Stieltjes integral, product measures and multiple integrals, Tonelli and Fubini theorems, convergences (pointwise, in measure, almost uniform), signed and complex measures, Hahn, Jordan, and Lebesgue decompositions (absolutely continuous and singular measures), Hardy-Littlewood maximal function, Lebesgue differentiation theorem, Radon-Nikodym derivative, Lp spaces and their duality. Additional topics depending on time: bounded variation functions, absolutely continuous functions, Hilbert space theory, Fourier series in L2, Borel measures in (locally) compact spaces and Riesz representation theorem.

B1: FUNCTIONAL ANALYSIS

Spaces with norms, Banach spaces, separability, Riesz lemma and compactness of closed balls, uniformly convex norms, distance from a point to a convex closed set, spaces with inner product, Hilbert spaces, orthogonal complement, projections, orthonormal sets and bases, Bessel's inequality, Riesz-Fischer theorem, Parseval identity, separability and Schmidt's theorem, dual space, Riesz theorem for Hilbert spaces, Hahn-Banach theorem (and Bohnenblust-Sobczyk theorem in its general form), second dual, reflexivity, Baire category theorem, uniform bounded operators, algebra B(X), adjoint operator, principle of uniform boundedness, convergence (in norm, strong, weak), open mapping theorem, closed graph theorem, spectrum of an operator (point spectrum, continuous spectrum, residual spectrum), compactness of spectrum, holomorphs of operator, spectral radius, compact operator, self-adjoint operator, spectral theorem for compact self-adjoint (and normal) operators. Additional topics depending on time: locally convex spaces, Frechet spaces, Schauder theorem for compactness of operator and its dual, spectra of compact operators, Riesz-Schauder theorem, Hamel and Schauder bases, fixed point theorems, von Neumann ergodic theorem, Hilbert-Schmidt operators, extreme points, Krein-Milman theorem, and integral representations.

B2: COMPLEX ANALYSIS

Topology of the complex plane (compact, connected sets), extended complex plane, limits and continuity of functions, series of numbers and functions (Weierstrass test), contour integrals, derivative and holomorphy, Cauchy-Riemann equations, special functions (exponential, branches of logarithm, powers, branches of roots, functions defined by contour integrals, functions defined by power series), Cauchy-Goursat theorem, existence of antiderivative in special sets (e.g., convex sets), local Cauchy formulas, Taylor and Laurent series, root multiplicity, singularities (poles, essential singularities and related criteria), Morera’s theorem, Cauchy estimates, Liouville's theorem, fundamental theorem of algebra, identity principle, maximum principle, open mapping principle, winding number, homotopy of curves, chains and cycles (of curves), homology, spherical Cauchy theorem, residue theorem (integration calculations), meromorphic functions, argument principle, Rouché's theorem, simple connectivity (homotopic, homological, topological), finite connectivity, polynomial approximation and existence of a branch of logarithm in a simply connected set, function periods in a finitely connected set, uniform convergence on compact subsets of an open set, Montel's theorem, Hurewicz’s theorem, Schwarz lemma, Riemann's theorem, conformal automorphisms of the disk and the upper half-plane. Additional topics depending on time: Runge's approximation theorem, Jensen's formula and inequality, entire functions (canonical representation), Weierstrass factorization theorem, Mittag-Leffler theorem, harmonic functions (Poisson's formula, harmonic conjugate, boundary values, reflection principle, Harnack's principle, Dirichlet problem), Γ and ζ functions, prime number theorem, analytic continuation and monodromy theorem, Hp spaces.

B3: HARMONIC ANALYSIS

Fourier series in L1 (and hence in Lp), Riemann-Lebesgue lemma, convergence criteria (Dini, Jordan), Dirichlet kernels, Fejér kernels, Poisson kernels, approximations of the identity, summability in norm, density of trigonometric polynomials in Lp, Fourier transform in L1, Riemann-Lebesgue lemma, convergence criteria (Dini, Jordan), Fourier transform in Schwarz space and in the space of tempered distributions (e.g., in the space of complex Borel measures), Fourier transform in L2, Hausdorff-Young inequality, Fourier transform in Lp ($1<p<2$), bounded linear operators in L1 and L2 invariant under translations, inversion formula in L1 and L2, Gaussian kernels, Poisson kernels, strongly and weakly bounded linear operators in Lp, maximal operator of a family of operators, Marcinkiewicz theorem, reiteration of Hardy-Littlewood maximal operator (in Lp and in LlogL) and its role in controlling other maximal operators, Calderón-Zygmund decomposition, conjugate Poisson kernel and Hilbert operator in Lp for dimension 1, multipliers, pointwise inversion of the Fourier transform (and pointwise convergence of Fourier series) for dimension 1, Fourier transform of positive measures, positive definite distributions, Bochner's theorem. Additional topics depending on time: BMO space, John-Nirenberg theorem, reverse Hölder inequalities, Hilbert operator in L^∞ and BMO, Fourier transform Lp → Lp' is not onto (van der Corput lemma and Khintchin inequality), singular integrals and Calderón-Zygmund operators, Riesz operators.

B4: ERGODIC THEORY

Examples of measurable dynamical systems, Poincaré's recurrence theorem, Von Neumann and Birkhoff ergodic theorems, Von Neumann theorem for squares, application in Ramsey theory (Furstenberg-Sárközy theorem), applications (equidistribution of sequences, normal numbers, continued fractions, strong law of large numbers for stationary stochastic processes), examples of weak mixing systems and equivalent definitions, strong mixing, isomorphism, factors, Kronecker factor, Halmos-Von Neumann discrete spectrum theorem, invariant measures in compact metric spaces, uniquely ergodic systems, equidistribution of irrational polynomials, ergodic analysis of invariant measures, multiple ergodic theorem of Furstenberg, application in Ramsey theory (Roth's theorem), entropy of partition and dynamical system, calculation in simple cases, non-isomorphism of Bernoulli 2-shift and 3-shift, Shannon-McMillan-Breiman theorem.

Γ0: RIEMANNIAN GEOMETRY

I. **Differentiable Manifolds**: Differentiable manifolds and mappings. The tangent space and the tangent bundle. Submanifolds. Vector fields and Lie derivative. Integration of vector fields and flows.
II. **Connections on Manifolds**: Linear connections. Geodesics and the exponential map.
III. **Riemannian Manifolds**: Riemannian metrics. Levi-Civita connection. Geodesics and normal coordinates on Riemannian manifolds. Geodesics in model spaces.
IV. **Geometry and Distance**: Distance and topology on a Riemannian manifold. Completeness and the Hopf-Rinow theorem. Isometries and the Myers-Steenrod theorem.
V. **Curvature**: The curvature tensor. Sectional curvature and Ricci curvature. Riemannian submersions and O'Neill's formulas. The second Bianchi identity, Schur's theorem, and Einstein manifolds.
VI. **Geometry and Topology**: The Jacobi differential equation. Conjugate points and the Cartan-Hadamard-Kobayashi theorem. Spaces of constant sectional curvature and classification. Variation of the length functional and Synge's theorem. The Bonnet-Myers theorem.

C1: DIFFERENTIABLE MANIFOLDS

I. **Differentiable Manifolds**: Differentiable manifolds. Smooth mappings on manifolds. Quotient manifolds.
II. **Tangent Space**: Tangent space. Tangent bundle. Submanifolds. Constant rank theorems. Tangent bundle. Partition of unity. Vector fields.
III. **Elements of Lie Groups**: Lie groups. Lie algebras.
IV. **Differential Forms**: Differential 1-forms. Differential k-forms. Exterior derivative. Lie derivative and interior multiplication.
V. **Integration**: Orientations. Manifolds with boundary. Integration on manifolds. Stokes' theorem.
VI. **DeRham Theory**: DeRham cohomology. Long exact sequence in cohomology. Mayer-Vietoris sequence. Homotopy invariance. Cohomology computations.

C2: ALGEBRAIC TOPOLOGY – HOMOTOPY

I. **Homotopy**: Homotopy mappings. Homotopy type. Categories, functors, and algebraic invariants. Path-connected components.
II. **The Fundamental Group**: Construction of the fundamental group. Examples and applications. Free groups and free products. The Seifert-Van Kampen theorem. Calculations and applications of the Seifert-Van Kampen theorem.
III. **Covering Spaces**: Basic concepts and examples. Lifting mappings to covering spaces. Covering mappings and the fundamental group. The universal cover of a space. Automorphisms of a covering. Classification of coverings of a space via subgroups of the fundamental group.
IV. **Higher Homotopy Groups**: H-groups and loop spaces. Suspension. Homotopy groups. Exact sequences. Fibrations. The role of the base point. The homotopy groups of spheres.

C3: ALGEBRAIC TOPOLOGY – HOMOLOGY

I. **Homology**: Homotopy and homotopy mappings. Homotopy type. Categories, functors, and algebraic invariants. The singular homology groups of a topological space. Chain complexes and exact sequences. The Eilenberg-Steenrod axioms for a homology theory and consequences. Singular homology. The homotopy axiom for singular homology. The excision axiom for singular homology. The Mayer-Vietoris exact sequence and applications of singular homology. The Hurewicz theorem.
II. **Homology with Coefficients**: The tensor product. The torsion product. Universal Coefficient Theorem. Singular homology with coefficients.
III. **Cohomology**: Homomorphism groups. Hom and Ext. Cohomology of chain complexes. Singular cohomology.
IV. **Products**: The cross product, the Eilenberg-Zilber theorem, and the Kunneth formula. The cross product in cohomology. The cup product and applications.
V. **Topological Manifolds and Duality**: Orientation of topological manifolds. The singular homology of a topological n-manifold in degrees ≥n. The cap product. Algebraic limits. Poincare-Lefschetz duality. Applications.

C4: GEOMETRY OF DYNAMIC SYSTEMS

Differential equations and integration of vector fields. Flows and the nature of their trajectories. Invariant and minimal sets. Linear dynamic systems. Circle rotations. One-parameter subgroups of the n-torus. Gradient vector fields. The shift in the space of sequences from a finite alphabet. Invariant measures. Unique ergodicity. Weyl's equidistribution theorem. Solenoids and additive machines. Conjugacy. The logistic map. The Smale horseshoe. Chaos. Sharkovskii's theorem. Poincare-Bendixson theory and applications. Flows on the 2-torus without fixed points. Circle homeomorphisms and Poincare's rotation number. Semi-conjugacy with rotations. The Denjoy-Koksma inequality and unique ergodicity. Denjoy's theorem and A. J. Schwartz's theorem. Examples of C^1 diffeomorphisms by Denjoy-Herman.

D10: PARTIAL DIFFERENTIAL EQUATIONS

Laplace's Equation: Basic properties of harmonic functions, Harnack's inequality, Fundamental solution, Green's functions, Dirichlet kernel for ball and half-space, Poisson's equation, Maximum principle, Energy methods, Dirichlet's principle, Perron's method.

Heat Equation: Fundamental solution, Cauchy problem, Inhomogeneous problem, Mean value property, Harnack's inequality, Maximum principle, Smoothness, Energy methods.

Wave Equation: Kirchhoff and Poisson formulas, Inhomogeneous problem, Energy methods.

Elliptic Equations: Maximum principle, A priori estimates, Elements of Functional Analysis, Schauder estimates, Dirichlet problem in general case.

Parabolic Equations: Maximum principle, A priori estimates.

Nonlinear First-Order Equations: Method of characteristics.

D11: PARTIAL DIFFERENTIAL EQUATIONS – THEORY OF WEAK SOLUTIONS

Sobolev Spaces: Weak derivatives, Sobolev spaces, Properties, Approximation by smooth functions, Extension, Trace, Sobolev inequalities, Morrey inequalities, Compactness, Dual spaces, Spaces with time.

Elliptic Equations: Weak solutions, Existence of weak solutions, Energy methods, Fredholm alternative, Interior regularity, Boundary regularity, Eigenvalues, Eigenfunctions.

Parabolic Equations: Weak solutions, Galerkin method, Energy estimates, Existence of weak solutions, Uniqueness of weak solutions, Regularity of weak solutions.

Hyperbolic Equations: Weak solutions, Galerkin method, Energy estimates, Existence of weak solutions, Uniqueness of weak solutions, Regularity of weak solutions.

First-order Hyperbolic Systems: Weak solutions, Viscosity method, Energy estimates, Existence and uniqueness.

D12: ORDINARY DIFFERENTIAL EQUATIONS AND DYNAMIC SYSTEMS

Local existence of solutions (Picard-Lindelöf and Peano). Uniqueness of solutions. Gronwall's lemma. Smooth dependence of solutions on data and parameters.

Linear Systems: Fundamental solutions, constant and non-constant coefficients, asymptotic behavior of solutions.

Asymptotic Behavior of Nonlinear Equations: Stability and instability of solutions. Linearization. Lyapunov functionals for studying stability.

Poincare-Bendixson: Existence of periodic solutions. Elements of bifurcation theory in one and two dimensions. Phase diagrams for autonomous systems.

D14: CALCULUS OF VARIATIONS

Direct Methods of the Calculus of Variations: Existence of minimizers, Coercivity, Lower semicontinuity, Weak solutions of the Euler-Lagrange equations, Convexity, Systems, Quasi-convexity, Local minimizers, Constraints, Compensated compactness, Concentration compactness, Limiting cases of the Palais-Smale condition, Invariants, Noether's theorem, Pohozaev results, Brezis-Nirenberg, Lions, Struwe, Topology effects, Isoperimetric inequalities.

D15: MATHEMATICAL THEORY OF FLUIDS

I. **Navier-Stokes Equations for Incompressible Fluids**
II. **Basic Functional Spaces**: Inequalities and embedding theorems. Riesz and Leray-Schauder theorems.
III. **Nonlinear Static Case**: Weak form of the problem. Existence and uniqueness of the solution. Classical solution.
IV. **Nonlinear Non-Static Case**: Weak form of the problem. Global and local solutions. Galerkin method. Existence and uniqueness of the global solution for n=2. Existence and uniqueness of the local solution for n=3. Classical solution. Existence of the global weak solution for n=3.
V. **Brief Mention of the Navier-Stokes Equations for Compressible Fluids**, Euler and Prandtl equations.

H10: NUMERICAL ANALYSIS

I. Norms and Inner Products in a Linear Space. Cauchy-Schwarz inequality. Norm induced by an inner product. Basic norms of vector spaces, such as the Euclidean norm, the maximum norm, the sum norm, the Frobenius norm, and the p-norm. Young's, Hölder's, and Minkowski's inequalities. Convergence in normed spaces and completeness of normed spaces. Equivalence of norms. Equivalence of norms in finite-dimensional spaces. Best approximations from a subspace in an inner product space. Matrix norms. Submultiplicative and natural matrix norms. Characterization of natural matrix norms induced by the maximum norm, the sum norm, and the Euclidean norm. Invertibility of the matrix I−AI - AI−A and representation of the inverse of I−AI - AI−A as a geometric series of AAA.
II. Linear Systems: Condition number of a matrix. Perturbation analysis for linear systems. Influence of truncation and rounding errors on the solution of linear systems. Gaussian elimination and PA=LUPA = LUPA=LU decomposition of a matrix. Cholesky decomposition for Hermitian and positive definite matrices. Iterative methods: Gauss-Seidel, Jacobi, SOR. General convergence theory of iterative methods. Positive definite matrices and their properties. Steepest descent method and its convergence. Construction of the conjugate gradient method and its convergence.
III. Approximating Solutions to Nonlinear Systems: Banach fixed-point theorem. Differentiable functions of several variables. Mean value theorem and quadratic approximation for differentiable functions. A general iterative method for approximating the root of smooth functions of one variable, its convergence conditions, and its order of convergence. Newton's method for approximating the solution of functions of one variable and systems of nonlinear equations. Convergence of Newton's method.
IV. Interpolation and Approximation: Polynomial interpolation (Lagrange, Hermite) and their approximative properties. Chebyshev polynomials. Spaces of piecewise polynomial functions (splines): construction and their approximative properties. Peano kernel theorem.
V. Numerical Integration: Orthogonal polynomials. Newton-Cotes rules. Gauss-Legendre rules. Error estimates of simple and composite numerical integration rules. Romberg's method. Numerical integration in two-dimensional polygonal domains.

H11: NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

1) Initial value problems for SDE systems. Uniqueness of the solution under the Lipschitz condition and under the one-sided Lipschitz condition. Gronwall's inequality.

(a) Monobasic methods.

(i) Euler's method: construction, stability, consistency, convergence. Euler's entangled method: existence and uniqueness of approximations, consistency, stability and convergence.

(ii) General theory of Runge-Kutta methods: stability, consistency, convergence. Examples of Runge-Kutta methods and multivariate methods.

(b) Multivariate methods: stability, consistency, convergence. Examples of multivariate methods.

(c) Absolute stability, A-stability and stability function for Runge-Kutta methods; B-stability and algebraic stability of Runge-Kutta methods; G-stability of multivariate methods.

(d) Continuous and discontinuous Galerkin methods for initial value problems for SDE systems.

2) Boundary value problems. Finite differences for two-point problems. Stability, consistency and convergence of finite difference methods for the two-point problem.

3) Initial and boundary value problems for partial differential equations.

(a) Finite difference methods for the heat equation. Euler's direct and braided methods. The Crank-Nicolson method.

b) The transport equation. upwind and downwind methods. The Lax-Wendroff method. Computational dependency spaces. Stability and convergence. von Neumann stability.

(c) Second order hyperbolic equations. The wave equation. Finite difference methods.

(d) Finite difference methods for the elliptic equation in two dimensions. Methods of solving the symmetric by blocks and positive of certain matrices. Convergence of methods.

H12: FINITE ELEMENT METHODS

I. A brief introduction to Hilbert space theory. Riesz representation theorem. Lax-Milgram theorem. Galerkin's theorem. Generalized derivatives. Sobolev spaces. Density theorems on Lp spaces and Sobolev spaces.

II. Polynomial spaces of multivariable function spaces. Finite element spaces based on piecewise polynomial functions. Finite element spaces equivalent to displacement. The Bramble-Hilbert entry. Approximation properties of finite element spaces with approximation error estimates on norms of Sobolev spaces.

III. A weak formulation of the boundary value problem: a) for the second-order, linear two-point problem and b) for second-order, linear elliptic differential equations with partial derivatives. Construction of approximations of the weak solution by the finite element method. and error estimates for the finite element method.

IV. Construction of finite element methods for initial value and boundary condition problems for the heat equation and the wave equation.

H13: NUMERICAL LINEAR ALGEBRA AND OPTIMIZATION

I. Gauss elimination (partial and total driving). LU analysis. Cholesky analysis. Numerical solution of sparse systems. Backward error analysis.

II. General theory of linear least squares problem. QR analysis. Householder and Givens transformations. Specific value distribution (SVD) analysis. Calculation of eigenvalue analysis.

III. Generalized iterative method. Jacobi and Gauss-Seidel methods. Relaxation methods (SOR, SSOR). Chebyshev methods. Maximum gradient descent method and method of conjugate gradients. Krylov subspace methods (Arnoldi, GMRES, QMR, QMR, MINRES).

IV. Pre-regulation techniques.

V. Approximation of eigenvalues and eigenvector matrices.

VI. Optimization methods for nonlinear unconstrained problems

a. Newton, steep descent methods with line search criteria

b. Quasi-Newton and conjugate gradient methods

VII. Optimization methods for non-linear constrained problems (CST conditions)

a. Barrier and penalty methods

b. Augmented Lagrangian methods

E10: PROBABILITY THEORY

Construction of probability spaces, random variables, construction of stochastic processes, independence, mean, the probabilistic method in combinatorics and number theory, types of convergence (almost certain, quadratic mean, probability, distribution), 0-1 Kolmogorov's law, weak and strong law of large numbers, three series theorem, Khintchine's law of recurrent logarithm, applications of limit theorems, characteristic functions, the central limit theorem for independent and dependent random variables (Lindeberg condition), applications of limit theorems, bounded mean, (sub)-martingales, limit theorems and applications.

E11: STOCHASTIC ANALYSIS

Continuous contemplative ascensions. Movement Brown. Timers. Examples. Continuous martingales and basic properties. The Doob-Meyer expansion. Continuous quadratically integrable martingales. Construction theorems of Brownian motion. Properties of Brownian motion trajectories. The Ito integral with respect to continuous square integrable martingales and basic properties. Variable changes in the stochastic integral. Ito's formula and applications. Representations of continuous martingales with the help of Brownian motion. Probabilistic study of Laplace and heat differential equations. Common stochastic differential equations - Examples. Existence and uniqueness theorems of solution. Solution of special forms of differential equations.