Rotating vortex-antivortex dipoles in ferromagnets under spin-polarised current

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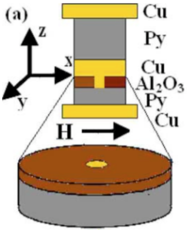
Injection of spin-polarized current through an aperture

Fixed layer: in-plane magnetization.

Free layer: elliptic element $250\,\mathrm{nm} \times 150\,\mathrm{nm}$.

Aperture to free layer: diameter $\sim 40\,\mathrm{nm}$.

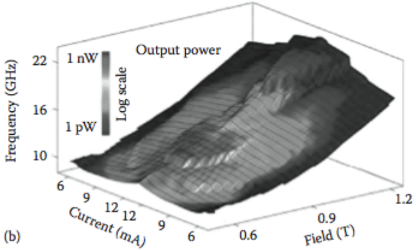


G. Finocchio et al, PRB 2008

Oscillations of the magnetization are measured ($\sim 1\,\mathrm{GHz}$).

Simulations show: they are due to spontaneous generation of vortex-antivortex (VA) pair with opposite polarities, in rotation.

Spin-transfer nano-oscillators

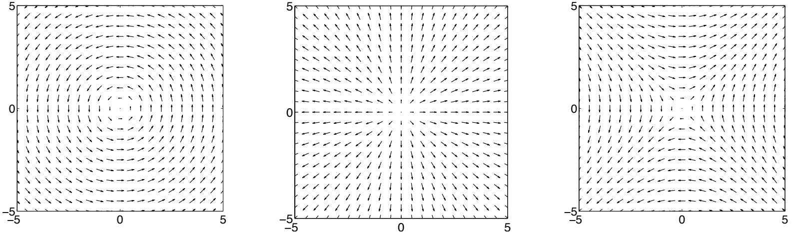


- simple (quasi-linear) dependence with magnetic field
- not-simple behavior with current
- jumps: more than one oscillation modes

Review in: Russek et al, "Spin-Transfer Nano-Oscillators" in "Handbook of Nanophysics"

A magnetic vortex

Vortex ($S = 1$) Vortex ($S = 1$) Antivortex ($S = -1$)



Vortex features

$S = \pm 1$ the winding number (a topological invariant)

$\lambda = \pm 1$ the vortex polarity

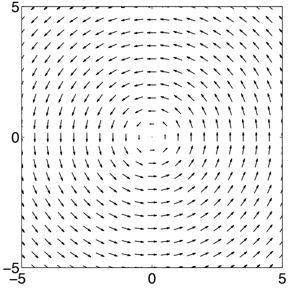
The skyrmion number $\mathcal{N}$

is a further topological invariant and it counts the number of times that the magnetization m covers the sphere $m^2 = 1$. For vortices:

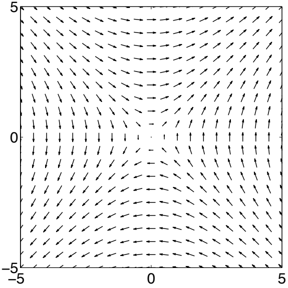
$$\mathcal{N} = \frac{1}{4\pi} \int n \, d^2x = -\frac{1}{2} S \lambda$$

$$n = \frac{1}{2} \epsilon_{\mu\nu} m \cdot (\partial_\nu m \times \partial_\mu m)$$:topological density

Vortex ($S = 1$, $\mathcal{N} = \pm 1/2$)



Antivortex ($S = -1$, $\mathcal{N} = \pm 1/2$)

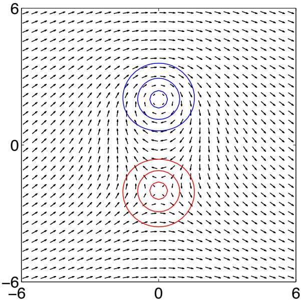


Vortex-antivortex dipole

Vortex: $S = 1, \quad \lambda = -1 \Rightarrow \mathcal{N} = \frac{1}{2}$

Antivortex: $S = -1, \quad \lambda = 1 \Rightarrow \mathcal{N} = \frac{1}{2}$

Vortex pair $\Rightarrow \mathcal{N} = 1$.



[S. Komineas, PRL 2007]

Magnetization vector:

$m = (m_1, m_2, m_3)$

Vector plot: (m_1, m_2)

Contour plot: m_3

Blue: $m_3 < 0$

Red: $m_3 > 0$

The model: Spin-torque in the LL equation

The polarized current-electrons exert a torque, modeled by an additional (Slonczewski) term in the Landau-Lifshitz (LL) equation:

$$\dot{m} = -m \times f + \alpha\, m \times \dot{m} - \beta\, m \times (m \times p)$$

$$f := \Delta m - m_3\, \hat{e}_3 + h_{\mathrm{ext}}$$

f: exchange + easy-plane anisotropy + external field

α: damping constant

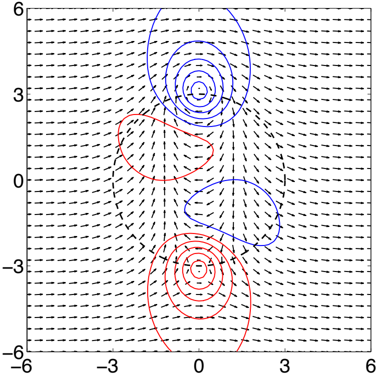
Spin polarization

$$\beta p = \beta(1,0,0), \qquad \beta < 0, \quad \text{proportional to current density.}$$

External field

$$h_{\mathrm{ext}} = (h_{\mathrm{ext}}, 0, 0).$$

Rotating vortex dipole under an aperture



Spin-polarized current through aperture of diameter $6\ell_{\mathrm{ex}}$ (dashed line).

Blue: polarity down

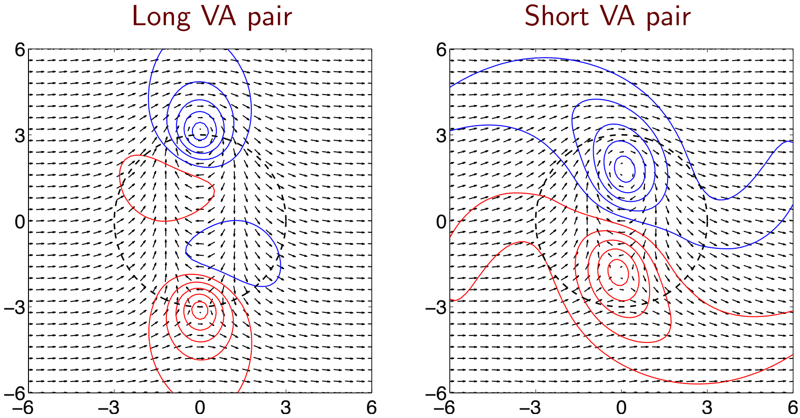
Red: polarity up

Simulation gives: Steady-state rotation.

Note that ground state is: $m_0 = (1, 0, 0)$.

Rotating vortex dipoles

$$\alpha = 0.02, \qquad \beta = -0.1, \qquad h_{\text{ext}} = 0.4$$

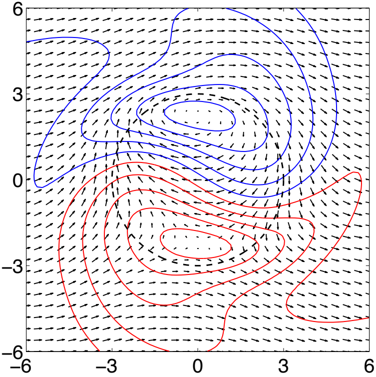


$\omega = 0.255$, $\omega = 0.539$

Rotating vortex dipoles

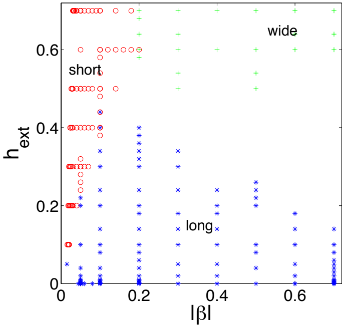
$$\alpha = 0.02, \qquad \beta = -0.2, \qquad h_{\text{ext}} = 0.6$$

Wide VA pair



$\omega = 0.820$

Phase diagram



stars: long VA pairs

circles: short VA pairs

crosses: wide VA pairs

- Small regions of overlap
- Long VA pairs for $h_{\rm ext} = 0$
- No steady-states for β too small
- Empty region: VA pairs plus satellite pairs rotating (similar to [Berkov, Gorn, PRB 2009])

The pure isotropic model

...that is, we assume exchange only (and external field):

$$f = \Delta m + h_{\rm ext}, \quad h_{\rm ext} = (h_{\rm ext}, 0, 0).$$

Use the sterographic projection of m from the point $m = (1, 0, 0)$:

$$X = \frac{m_2 + i m_3}{1 - m_1}.$$

Obtain equation of motion

$$(i - \alpha)\,\dot{X} = -4\,\partial_z\partial_{\bar{z}}X + \frac{8\overline{X}}{1 + X\overline{X}}\,\partial_z X\,\partial_{\bar{z}}X - (h_{\rm ext} - i\beta)\,X$$

where $z = x + iy$ the position on the complex plane, so $X{=}X(z, \bar{z})$.

- Expect precession around m_1 due to field $h_{\rm ext} = (h_{\rm ext}, 0, 0)$.

Rotating solutions

The simple form

$$X_0 = i\,\frac{z}{a_0}, \qquad a_0 : \text{complex constant} \qquad (\mathcal{N} = 1)$$

represents two merons: $m_3 = \pm 1$ at $z = \pm a_0$ [Gross, 1978], or a VA dipole.

Solution of the eqn of motion for $X(t=0) = X_0(z)$:

$$X(z,t) = i\,\frac{z}{a(t)}, \qquad a(t) = a_0 \exp\left(\frac{i\beta - h_{\rm ext}}{i - \alpha}\, t\right).$$

VA dipole in steady rotation for $\alpha h_{\rm ext} + \beta = 0$:

$$X(z,t) = i\,\frac{z}{a_0 e^{-i h_{\rm ext} t}}$$

It represents rotation of the vortex positions $\pm a(t) = \pm a_0 e^{-i h_{\rm ext} t}$ in the complex plane.

Virial relation (I)

An exact virial relation (of Derrick type) can be derived, for steady-state rotation, involving the frequency (ω) of rotation.

$$\omega \left(\ell + \frac{\alpha}{2} \int \epsilon_{\lambda\nu}\, x_\lambda x_\mu d_{\mu\nu}\, d^2x \right) = - \left(E_{\rm a} + h_{\rm ext}\, \mu_1 + \frac{\beta}{2} \int x_\mu \tau_\mu\, d^2x \right),$$

- Frequency of rotation ω
- Angular momentum: $\ell = \frac{1}{2} \int \rho^2 n\, d^2x \sim d_{\rm VA}^2$ (actually: $\ell \sim \mathcal{N}\, d_{\rm VA}^2$)
- Anisotropy energy $E_{\rm a}$ ($E_{\rm a} \approx \pi/2$ for single vortex)
- Total in-plane magnetization: $\mu_1 = \int (1 - m_1)\, d^2x$

Virial relation (II)

An exact virial relation (of Derrick type) can be derived, for steady-state rotation, involving the frequency (ω) of rotation.

A simplified form of the Derrick relation is

$$\omega \doteq -\left(\frac{E_{\mathrm{a}}}{\ell} + h_{\mathrm{ext}}\,\frac{\mu_1}{\ell}\right)$$

- Frequency of rotation ω
- Angular momentum: $\ell = \frac{1}{2}\int \rho^2 n\, d^2x \sim d_{\mathrm{VA}}^2$ (actually: $\ell \sim \mathcal{N}\, d_{\mathrm{VA}}^2$)
- Anisotropy energy E_{a} ($E_{\mathrm{a}} = \pi/2$ for single vortex)
- Total in-plane magnetization: $\mu_1 = \int (1 - m_1)\, d^2x$

Virial relation (III)

An exact virial relation (of Derrick type) can be derived, for steady-state rotation, which involves the frequency (ω) of rotation.

A simplified form of the Derrick relation is

$$\omega \doteq - \left(\frac{E_{\mathrm{a}}}{\ell} + h_{\mathrm{ext}} \, \frac{\mu_1}{\ell} \right)$$

- First term: rotation due to interaction between vortices
- Second term: rotation due to external field
- Both terms rely upon $\mathcal{N} \neq 0$ (i.e., $\ell \neq 0$).

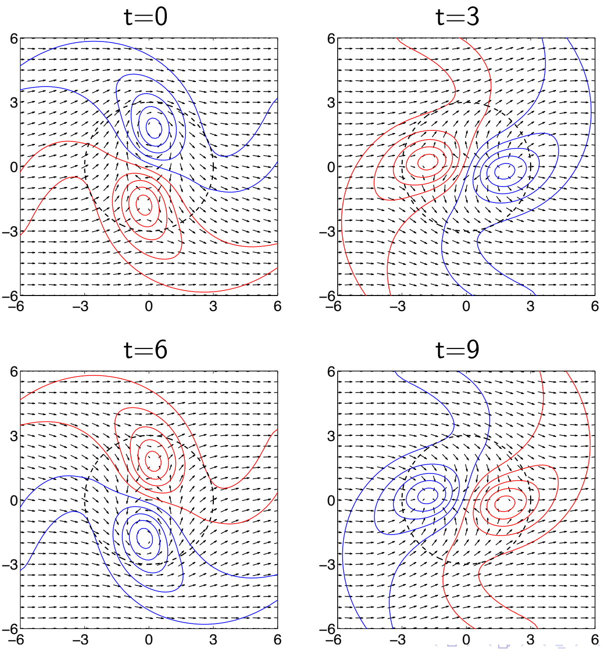
Asymptotics at large distances

Away from the vortex pair, we have Bessel eqns, so:

$$\frac{m_2 + i m_3}{1 + m_1} \sim \frac{1}{\sqrt{\rho}}\, e^{(i\lambda_1 - \lambda_2)\rho}, \qquad \rho \to \infty.$$

- $\lambda_2 \ll 1$: for $h_{\rm ext} \lesssim 0.01 \to$ VA pairs non-localized.
- $\lambda_2 \ll 1$: for β too small $\to$ VA pairs collapse at $\omega_{\max} \sim \sqrt{h_{\rm ext}(1 + h_{\rm ext})}, \qquad h_{\rm ext} \gtrsim 0.1.$
- $\lambda_1 \neq 0$: spiralling waves emanating from rotating pair.

A full rotation



Conclusions

- Vortices and antivortices can be generated by spin-polarized current.
- A vortex-antivortex dipole (with $\mathcal{N} = 1$) is rotating.
- Three (at least) vortex-antivortex modes: well-separated vortex-antivortex or two-merons.
- In-plane field $h_{\rm ext} = (h_{\rm ext}, 0, 0)$, typically expected to induce magnetization precession (around x-axis), is actually giving rotation of a configuration with $\mathcal{N} = 1$.
- Rotational motion is stabilized by the spin-polarized current.
- Rotation frequency can be tuned by current and external field.
- The magnetostatic field can be incorporated in the formalism (some simulations have been published).