# University of Crete
## Department of Mathematics and Applied Mathematics

## Logic Seminar

12:00, Friday, 12 July, 2019
Room B201

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*Compactness and measure in second order arithmetic*

Considerable research in subsystems of second order arithmetic concerns two weak forms of Koning's lemma:

- WKL: every infinite binary tree has a path.
- WWKL: every 'fat' (in the sense of measure) binary tree of has a path.

WKL is a compactness principle that is essentially equivalent to a number of well-known theorems in mathematics including the Brouwer fixed point and the Heine–Borel theorem. WWKL is essential in the formal development of measure theory, and essentially equivalent to the Vitali covering theorem and the monotone convergence theorem for Lebesgue measure in the unit interval. We investigate the strength of two principles between $\mathrm{WWKL}_0$ and $\mathrm{WKL}_0$, of a measure-theoretic character:

- $P$: every positive tree has a perfect subtree
- $P^+$: every positive tree has a positive perfect subtree.

Under $\mathrm{RCA}_0$ we have WKL $\rightarrow P^+ \rightarrow P \rightarrow$ WWKL. We show that all of these implications are strict, modulo $\mathrm{RCA}_0$. The model for the first non-implication involves forcing with perfect subsets of positive measure trees, avoiding completions of Peano Arithmetic. The last two non-implications are based on models generated by random reals (with respect to different measures). Our methods are largely combinatorial (weighted hypergraph transversals, hitting sets) and have interesting consequences in algorithmic randomness with regards to: (a) randomized algorithms producing algorithmically random strings; (b) probability of computing perfect or 'fat' trees of random reals. Our work answers questions from: [C. Chong, W. Li, W. Wang, Y. Yang. *On the computability of perfect subsets of sets with positive measure*. Proc. Amer. Math. Soc., 2019.] Limit groups have been introduced by Z. Sela in his celebrated solution of Tarski's problem (2001). These groups coincide with the long studied class of finitely generated fully residually free groups introduced by Baumslag in 1967. Limit groups have a natural interpretation in many mathematical disciplines: topology, geometric group theory, algebraic geometry and logic to name a few. This fact makes them an interesting class to study and also provides us with an abundance of tools to do so.

In this talk I will present all different points of view on limit groups and survey some basic results about them. Moreover, I will prove the existence of a natural group with interesting universal and homogeneous properties with respect to the class of limit groups, that resembles the relation between an algebraically closed field and the class of fields. This is joint work with O. Kharlampovich and A. Miasnikov.